MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

CONCEPTS FOR ADVANCED MATHEMATICS, C2

Practice Paper C2-B

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- There is an **INSERT SHEET** for question 11.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Find all the angles in the range $0^0 \le x \le 360^0$ satisfying the equation $\sin x + \frac{1}{2}\sqrt{3} = 0$. [3]

2 Solve the equation $3^x = 15$, giving your answer correct to 4 decimal places. [3]

3 The sum to infinity of a geometric series is 5 and the first term is 2.
Find the common ratio of the series. [3]

4 The first 3 terms of an arithmetical progression are 7, 5.9 and 4.8.

Find

(i) the common difference, [1]

(ii) the smallest value of n for which the sum to n terms is negative. [4]

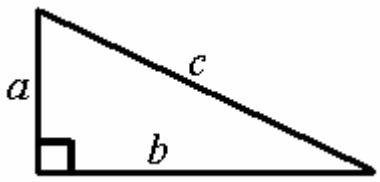
5 The gradient of a curve is given by the function $\frac{dy}{dx} = 2 - x$.
The curve passes through the point (1, 2).

Find the equation of the curve. [4]

6 Evaluate $\int_1^2 \left(x^2 + \frac{1}{x^2} \right) dx$. [5]

7 **(i)** Using the triangle, show that [3]

$\sin^2 x + \cos^2 x = 1$.



(ii) Hence prove that [2]

$1 + \tan^2 x = \frac{1}{\cos^2 x}$.

8 Draw two sketches of the graph of $y = \sin x$ in the range $0^o \le x \le 360^o$.

(i) On the first sketch, draw also a sketch of $y = \sin(2x)$. [2]

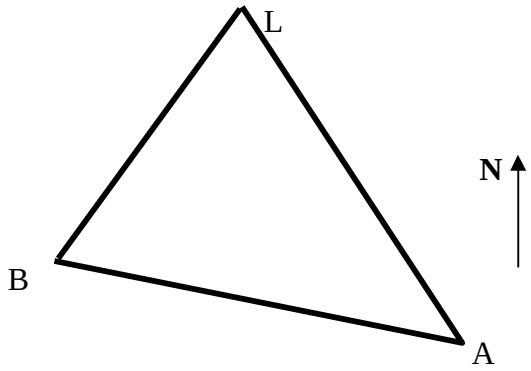
(ii) On the second sketch, draw also a sketch of $y = 2\sin x$. [2]

9 A sector of a circle has an angle of 0.8 radians. The arc length is 5 cm.
Calculate the radius of the circle and the area of the sector. [4]

Section B (36 marks)

10 At 1200 the captain of a ship observes that the bearing of a lighthouse is 340^o. His position is at A.
At 1230 he takes another bearing of the lighthouse and finds it to be 030^o. During this time the ship moves on a constant course of 280^o to the point B.

His plot on the chart is as shown in Fig. 11 below.



Not to scale

Fig. 11

(i) Write down the size of the angles LAB and LBA. [2]

(ii) The captain believes that at A he is 5 km from L. Assuming that LA is exactly 5 km, show that LB is 4.61 km , correct to 2 decimal places, and find AB.
Hence calculate the speed of the ship. [8]

(iii) The speed of the ship is actually 10 kilometres per hour. Given that the bearings of 340^o and 030^o and the ship's course of 280^o are all accurate, calculate the true value of the distance LA. [2]

11 ***You should use the insert sheet for this question.***

John records the speed of a car in metres per second over a period of 10 seconds. His results are shown in the table below.

t	0	2	4	6	8	10
v	0	9	15	18	15	10

(i) The speed-time graph on the insert sheet provides the axes and the first two points plotted. Plot the remainder of these points and join them with a smooth curve. [2]

The area between this curve and the t-axis represents the distance travelled by the car in this time.

(ii) Using the trapezium rule with 6 values of t estimate the area under the curve to give the distance travelled. Illustrate on your graph the area found. [3]

(iii) John's teacher suggests that the equation of the curve could be $v = 6t - \frac{1}{2}t^2$.

Find, by calculus, the area between this curve and the t axis. [5]

(iv) Plot this curve on your graph. Comment on whether the estimates obtained in parts (ii) and (iii) are overestimates or underestimates. [2]

12 Fig. 12 shows a window. The base and sides are parts of a rectangle with dimensions $2x$ metres horizontally by y metres vertically. The top is a semicircle of radius x metres. The perimeter of the window is 10 metres.

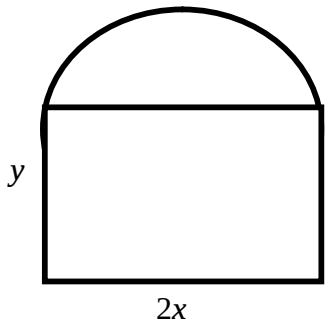


Fig.12

(i) Express y as a function of x. [2]

(ii) Find the total area, A m^2, in terms of x and y. Use your answer to part (i) to show that this simplifies to

$$A = 10x - 2x^2 - \frac{1}{2}\pi x^2 .$$ [4]

(iii) Prove that for the maximum value of A, $y = x$ exactly. [6]

NAME OF CANDIDATE:…….

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11 Speed-time graph with the first two points plotted.